Advanced Riemann Solvers

- Many codes use local Lax-Friedrichs and HLLE
- HLLC: Captures contact in hydrodynamics
- HLLD: Captures contact and Alfvén in MHD
- Better solvers \rightarrow less diffusion
- Method (Pons et al. 1998, Antón et al. 2006):
- Transform into orthonormal frame at each face
- Use SR Riemann solvers
- Transform fluxes back



Magnetized blast wave in sinusoidal coordinates, showing gas pressure and magnetic field, computed using HLLD. The solution matches that obtained in Minkowski coordinates.

Constrained Transport

- Keeps $\nabla \cdot \vec{B} = 0$, natural discretization of Stokes' law
- Formulated in GR (Evans & Hawley 1988)
- Staggered mesh: \vec{B} defined on faces
- Implementation proven in Athena (Gardiner & Stone 2005, 2008)



Schematic showing how \vec{E} is calculated on an edge (center) self-consistently with the Riemann fluxes. Fields at faces and cell centers are differenced to calculate gradients, which are upwinded back to faces. These contribute to calculating the edge values.

Athena++: A New GRMHD Code for Black Hole Accretion

Christopher J. White and James M. Stone Princeton University

Summary

- Extending Athena to general relativity, while maintaining its advantages in **speed** and **accuracy** for ideal MHD.
- Goal: studying **radiation-dominated** accretion flows near black holes.

Convergence Tests

- 2nd-order (piecewise linear) spatial reconstruction
- 2nd-order timestepping (van Leer integrator)
- Tested with linear waves
- Vector of primitives *P* obeys $\partial_t P + A \partial_x P = 0$
- Perturb background with eigenvectors of *A*
- Done in coordinates with time-space terms in metric



Hydrodynamics convergence tests. For waves propagating slower than the sound speed, HLLE is more diffusive and less accurate than HLLC.



MHD convergence tests. Again a better Riemann solver leads to lower errors, by a factor of almost 5 in the entropy wave case.



Performance

- Key goal: fast and scalable
- Zone updates per second
- 2.5 GHz Ivybridge, single core

			SR	GR
	Hydro	HLLE	1,500,000	1,000,000
		HLLC	1,500,000	1,000,000
	MHD	HLLE	490,000	300,000
		HLLD	150,000	100,000
r performance improvement expected				

• Further performance improvement expected



Density in magnetized Fishbone-Moncrief (1976) torus around Schwarzschild black hole. The torus has less azimuthal velocity than needed to resist infall, and it has been given a poloidal magnetic field. This shows the beginning of accretion at t = 30M.

Looking Forward

- Algorithms
 - Adaptive mesh refinement
 - Radiative transfer
- Applications
 - Bardeen-Petterson effect
 - Tidal disruption