Advanced Riemann Solvers
- Many codes use local Lax-Friedrichs and HLLE
- HLLC: Captures contact in hydrodynamics
- HLLD: Captures contact and Alfvén in MHD
- Better solvers → less diffusion
  - Transform into orthonormal frame at each face
  - Use SR Riemann solvers
  - Transform fluxes back

Constrained Transport
- Keeps $\nabla \cdot \vec{B} = 0$, natural discretization of Stokes’ law
- Formulated in GR (Evans & Hawley 1988)
- Staggered mesh: $\vec{B}$ defined on faces
- Implementation proven in Athena (Gardiner & Stone 2005, 2008)

Summary
- Extending Athena to general relativity, while maintaining its advantages in speed and accuracy for ideal MHD.
- Goal: studying radiation-dominated accretion flows near black holes.

Convergence Tests
- 2nd-order (piecewise linear) spatial reconstruction
- 2nd-order timestepping (van Leer integrator)
- Tested with linear waves
  - Vector of primitives $P$ obeys $\partial_t P + A \partial_x P = 0$
  - Perturb background with eigenvectors of $A$
  - Done in coordinates with time-space terms in metric

MHD convergence tests. Again a better Riemann solver leads to lower errors, by a factor of almost 5 in the entropy wave case.

Performance
- Key goal: fast and scalable
- Zone updates per second
  - 2.5 GHz Ivybridge, single core
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Looking Forward
- Algorithms
  - Adaptive mesh refinement
  - Radiative transfer
- Applications
  - Bardeen-Petterson effect
  - Tidal disruption

Density in magnetized Fishbone-Moncrief (1976) torus around Schwarzschild black hole. The torus has less azimuthal velocity than needed to resist infall, and it has been given a poloidal magnetic field. This shows the beginning of accretion at $t = 30M$. 

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Magnetized blast wave in sinusoidal coordinates, showing gas pressure and magnetic field, computed using HLLD. The solution matches that obtained in Minkowski coordinates.

Schematic showing how $\vec{E}$ is calculated on an edge (center) self-consistently with the Riemann fluxes. Fields at faces and cell centers are differenced to calculate gradients, which are upwinded back to faces. These contribute to calculating the edge values.